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A note on non-commutative orbifold field theories

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Abstract: We suggest that orbifold field theories which are obtained from non-commutative $\mathcal{N} = 4$ SYM are UV finite. In particular, non-supersymmetric orbifold truncations might be finite even at finite values of N_c.

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Contents

1. Introduction

Recently non-commutative Yang-Mills theory [1] attracted a lot of attention, mainly due to discoveries of new connections to string theory [2, 3, 4]. In a recent paper [5], it was suggested that the divergences of the non-commutative Yang-Mills theory are dictated by the large N_c limit of the theory. Namely, that divergences occur only in planar diagrams (the observation that the planar commutative and non-commutative theories are the same up to phases in the Green functions, was already made in [6]. A careful analysis of the divergences was carried out in ref. [7]).

Another direction of research is the study of orbifold field theories — motivated by the AdS/CFT correspondence [8]. It was conjectured by Kachru and Silverstein [9] that orbifolds of $AdS_5 \times S^5$ which act on the S^5 part define a large N_c finite theory, even when the R-symmetry is completely broken and the theory is not supersymmetric. This conjecture was later proved using both field theory [10] and string theory [11] techniques.

In this note we would like to consider non-commutative Yang-Mills theories which are obtained by an orbifold truncation of $\mathcal{N} = 4$ SYM. We suggest that these theories are UV finite, namely that there are no divergent Feynman diagrams, even when the theory under consideration is non-supersymmetric and the number of colors is finite. Note, however, that our conjecture relies on recent suggestions [5, 12] about UV finiteness of non-planar graphs in the non-commutative theory. The later were not fully proved yet, but seems to be necessary if the non-commutative theory is renormalizable.

2. Orbifold field theories

Orbifold field theories are obtained by a certain truncation of a supersymmetric Yang-Mills theory. Let us consider the special case of $\mathcal{N} = 4$. The truncation procedure is as follows: consider a discrete subgroup Γ of the $\mathcal{N} = 4$ R-symmetry group SU(4). For each element of the orbifold group, a representation γ inside $\mathrm{SU}(|\Gamma|N_c)$ should be specified. Each field Φ transform as $\Phi \to r\gamma^\dagger\Phi\gamma$, where r is a representation matrix inside the R-symmetry group. The truncation is achieved by keeping invariant fields. The resulting theory has a reduced amount of supersymmetry, or no supersymmetry at all. It was conjectured [9], based on the AdS/CFT conjecture, that the truncated large N_c theories are finite as the parent $\mathcal{N} = 4$ theory. Later it was proved [10] that the planar diagrams of the truncated theory and parent theories are identical. In particular it means that the perturbative beta function of the large N_c daughter theory is zero and that the theory is finite.

In the cases of $\mathcal{N} = 2$ truncations, there is only one-loop (perturbative) contribution to the beta function. Its vanishing indicates the perturbative finiteness of the daughter theory at finite N_c as well.

In $\mathcal{N} = 1$ truncations the situation is more subtle. Indeed, the theory is finite at finite values of N_c, but the finiteness is due to Leigh-Strassler type of arguments [13]. In that case one should consider the $\mathrm{SU}(N_c)$ version of the theory (and not the $\mathrm{U}(N_c)$ theory which is obtained from the string theory orbifold), since the U(1) beta function is always positive at the origin. In addition $1/N_c^2$ shifts of the Yukawa couplings are needed [14].

In the non-supersymmetric case there are no known examples of finite theories at finite N_c, though attempts in this direction using orbifolds were recently made [15]. As we shall see, due to non-commutativity such examples can be found.

3. Perturbative behavior of non-commutative Yang-Mills theories

Recently, several authors [7, 5] analyzed the renormalization behavior of non-commutative Yang-Mills theories (for an earlier discussion see [6]. Related works are [16, 17]). We briefly review their analysis. The Feynman rules of the commutative and non-commutative theories in momentum space are very similar. In fact the only difference is that each vertex of the non-commutative theory acquires a phase, $\exp i\Sigma_{i<j} k_i \wedge k_j$ (the Moyal phase), with respect to the vertex of ordinary commutative theory [6]. For planar diagrams, this phase cancels at internal loops and the only remnant is an overall phase. Therefore the planar commutative and non-commutative theories are similar, in accordance with recent findings [18, 19, 20]. In particular, thermodynamical quantities such as the free energy and the entropy are not affected by non-commutativity at the planar limit [19].

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Another claim [5, 7] is that the oscillations of the Moyal phases at high momentum would regulate non-planar diagrams, namely that UV divergences of non-planar diagrams would disappear. There are two exceptional cases in which non-planar diagrams will still diverge [7]: (i). When the non-planar diagrams consist of planar sub-diagrams which might diverge. (ii). For specific values of momentum (a zero measure set) the Moyal phase can be zero. Indeed, it was shown lately [12] that in theories which contain scalars - there are non-planar divergences. The theories that we will consider contain scalars and therefore contain infinities, however [12] interpret these divergences as *infra-red* divergences. The reason is as follows: The integrals which are associated with non-planar graphs converge unless the Moyal phase is set to zero by a vanishing incoming momentum. In this case new type of divergences appear, and it seems that infinite number of counterterms are needed. Therefore the non-commutative theory seems to be non-renormalizable. However, the authors of ref. [12] suggest that the re-summation of these divergent non-planar graphs would yield a finite result. Their observation is based on the similarity between the present case and the standard (commutative) IR divergences. Note that infra-red divergences are, anyways, expected in theories with massless particles. Thus, 'truly' divergences in the non-commutative theory occur only in planar graphs.

4. Finiteness of non-commutative orbifold field theories

Let us now consider an orbifold truncation of non-commutative $\mathcal{N} = 4$ SYM. These theories can be defined perturbatively by a set of Feynman rules. The natural definition would be to attach to each vertex the corresponding Moyal phase [6]. According to ref. [5], the only potential divergences are the ones which arise in planar diagrams. Non-planar diagrams are expected to be finite, except for some "accidental" divergences [5]. Note that the infinities which do occur in these graphs are associated with the infra-red [12] and therefore do not contradict our claim that orbifold field theories are UV finite.

According to the analysis of [10], the planar diagrams of an orbifold theory can be evaluated by using the corresponding diagrams of the parent non-commutative $\mathcal{N} = 4$. These diagrams are finite since they differ from the commutative $\mathcal{N} = 4$ only by an overall phase. In this way sub-divergences of non-planar diagrams will also be canceled. We therefore conclude that any orbifold truncation of non-commutative non-compact $\mathcal{N} = 4$ SYM is finite. In particular, it means that we might have a rich class of non-supersymmetric gauge theories which are finite, even at finite N_c (in contrast to ordinary non-supersymmetric orbifold field theories, where the two loop beta function is generically non-zero). Note that though these theories might be finite, they are certainly not conformal.

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Let us consider a specific example [21]. The example is an SU(N)$\times$SU(N) gauge theory with 6 scalars in the adjoint of each of the gauge groups and 4 Weyl fermions in the $(N, \bar{N})$ and 4 Weyl fermions in the $(\bar{N}, N)$ bifundamental representations. It is the theory which lives on dyonic D3 branes of type 0 string theory and can be also understood, from the field theory point of view, as a $\mathbb{Z}_2$ orbifold projection of $\mathcal{N} = 4$ SYM [22]. The large N_c commutative theory contains a line of fixed points. We suggest that its analogous non-commutative (finite N_c) theory is finite at that line. Namely, at $g_1 = g_2 = h_1 = h_2$, where g_1, g_2 are the gauge couplings and h_1, h_2 are the Yukawa couplings. Note that in contrast to the commutative case the position of the line of finite theories is *not* corrected by $1/N_c$ contributions.

Finally, it might be interesting to understand how the finiteness of these theories arise from string theory orbifolds.

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